Parameterization of Geometric Patterns of Islamic-Iranian Space Structure Domes; Case Study: Shabdari Arc

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Abstract

In this paper, parametric modeling of five types of common used motifs in Islamic-Iranian architecture including Hashte-Chahar-Lengeh, Hashte-Panj, Hashte-Bazoubandi, Chahar-Lengeh-AlmasTarash and Table Khoffte Rasteh are carried out. These motifs are selected because of low condensate of members and the ability to construct them appropriately as load barring rod elements in the dome space structure of slow Shabdari arch. Hence, motifs are projected on the geometry of slow Shabdari arch to create a dome space structure. The problem of parametric modeling of motifs is done in two cases of changing the number of basic modules and changing module production parameters. In the first case, the projection algorithm of basic modules depends on the dissemination coefficients representing the number of modules’ proliferation in horizontal and vertical direction of dome. In this case, the aim is to create sparse or delicate networks based on the form of fixed basic module. In the second case, the parameters used in the algorithm of basic modules themselves relies on the dissemination coefficients to produce new modules based on the considered motifs. Computer simulations for the second case of the problem of motifs’ parametric modeling show that, through a proper choice of the dissemination coefficients, slow Shabdari arch can be converted to common dome space structures such as lamella and ribbed domes.

Keywords: Islamic-Iranian Geometric motifs, Parametric Architecture, Slow Shabdari Arch, Dome Space Structures.
1. Introduction

Domes due to high structural efficiency as well as their benefits in terms of architecture are one of the best options for covering large openings. Cover of dome has a long history in Iran. Deficiency of firm and tall woods that are actually the main elements of flat cover led to the spread of dome cover and especially, it takes the place of flat cover at wider openings [1:5].

Traditional Iranian domes are prominent samples of building and Iranian architecture has created a variety of dome structures using traditional methods and yet many of them have remained stable even in earthquake prone zone of the country after centuries [2:747].

Nowadays, due to more diverse functions of architecture and the need to cover larger openings, the use of dome structure on its traditional form is not feasible and economic and the need for new technology is inevitable. Also, with the advancement of science and technology, new needs and demands are appeared in the field of structural engineering. The time factor becomes crucial in building structures and this has increased the tendency to prefabricated structures. However, the use of structural systems and modern technologies regardless of the values and culture of a country is a challenge that has attracted the attention of the scientific and practical community in host countries. Iran is not an exception. To resolve the problems caused by the mismatch between the values of the host country and new technologies, modern technologies should be passed with the localization filter. In this regard, it can be seen that Islamic-Iranian geometric motifs that many architectural monuments of our country are decorated by them, have high potential for using in structure of space frame structures instead of conventional modules that are used in today's world.

Hence, this article investigates the possibility of using Islamic-Iranian geometric motifs in the structure of dome space of slow Shabdari arch, as the elements having structural role, despite keeping alive the vernacular architecture patterns. For this purpose, the parametric modeling of five types of most widely used motifs that can be constructed and used as rod elements of the space structures are extracted. Generative parameters are defined to express the parametric model of each of the motifs that various models can be created by their variation. For example, the effect of the variation of parameters for one of the motifs, Hashte-Chahar-Lengeh, on the final form of the dome of slow Shabdari arch is projected.

The most important point that should be noted is that some types of common domed space structures like lamella and ribbed domes can be created by changing the productive parameters of this motif.

2. Islamic - Iranian geometric motifs

According to some scholars such as Glom Beck and Wilber, making geometric the design, structure and space is the unifying force of architecture in Timuri period [3:14-15]. However, utilizing geometric designs is not the exclusive privileges of Islamic art because such designs can be seen in almost all Western and Eastern traditional arts and in this regard, we can refer to the decoration of windows glasses of cathedrals of Gothic style. These geometric shapes are expanded in Islamic art with logical criterion and reach perfection [4:28]. Generally, decorations in Islamic art are divided into three parts of plant, geometric and inscription. In Iran, geometric decoration is called Girih and usually is a combination of Shamse and polygon tools that are in harmonious fusion with each other. Scholars like Ernest Diaz and Oktai Aslanapa and Seljuk Molayem have attributed the geometric motifs to nomadic Turks. But unlike their ideas, Upham Pope knows geometric style of Seljuk period or girih chini Iranian innovation not Turkish [3:20-25]. Girih are shaped based on certain rules by using straight lines. Girih is applicable on planar and curved surfaces and with a variety of different materials. Girih in different periods of history of Iran and other Islamic countries has a lot of types and species. Among the most important girihs in the geography area of Iran, we
can refer to the slow, fast and loose girih. In addition to the above classification, girihs can be classified based on the type of applied Shamse in them as the girihs of 6, 8, 10, 12 [6:326].

Geometry of motifs is separated into two distinct parts. The first refers to the base pattern with the primary and small role that due to the geometric transformations such as transition, periods and repetition has become the main area and the second is the logic or repetition way of base pattern. The main objective of scientists of geometry is discovering the geometric proportions and relations of the components of the base pattern and their repetition way in order to make possible their redrawing with the same basic proportion but at different scales [7:204]. Some of the well-known and widely-used girihs in Iranian architecture are presented in Table 1.

Table 1. Some widely used girihs in Islamic architecture

<table>
<thead>
<tr>
<th>Name of Girih</th>
<th>Available sample</th>
<th>Geometric Shape</th>
<th>Location used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hashte-Chahar-Lengeh</td>
<td><img src="image1" alt="Hashte-Chahar-Lengeh" /></td>
<td><img src="image2" alt="Hashte-Chahar-Lengeh" /></td>
<td>Shrine of Imam Reza (AS)</td>
</tr>
<tr>
<td>Hashte-Panj</td>
<td><img src="image3" alt="Hashte-Panj" /></td>
<td><img src="image4" alt="Hashte-Panj" /></td>
<td>Gohar Shad Mosque</td>
</tr>
<tr>
<td>Hashte-Bazoubandi</td>
<td><img src="image5" alt="Hashte-Bazoubandi" /></td>
<td><img src="image6" alt="Hashte-Bazoubandi" /></td>
<td>National Museum of Iran</td>
</tr>
<tr>
<td>Chahar-Lengeh-Almas Tarash</td>
<td><img src="image7" alt="Chahar-Lengeh-Almas Tarash" /></td>
<td><img src="image8" alt="Chahar-Lengeh-Almas Tarash" /></td>
<td>Seyed mosque in Isfahan</td>
</tr>
<tr>
<td>Table Khofte Rasteh</td>
<td><img src="image9" alt="Table Khofte Rasteh" /></td>
<td><img src="image10" alt="Table Khofte Rasteh" /></td>
<td>Temple in India</td>
</tr>
</tbody>
</table>
3. Slow Shabdari Arch

In geometrical definition, arch refers to the line or curved shape and in terms of architecture is a strip of vault that is on the entrance of a port [9:5]. Choosing appropriate arch for the loads exerted on the vault and dome in Iranian architecture is based on the construction logic and building static that is resulted from long years of practical experience and expertise. Arch are divided into two major types of lean and sharp. The head of sharp arch, as its name implies, is sharp and is created from the intersection of two curved arches, which is called "Zigzag arch". The head of the lean arch has crescent shape and is a part of oval [9:8].

An example of porter sharp arch that is usually used in the outer shell of domes is Shabdari arch. In Iranian architecture, Shabdari arch is usually used in the domes (not in vaults) because circle is used to draw them and drawing this arch in space for the arch, due to the difficulty of finding the center of the circle is almost impossible and because a kind of stencil is used in the outer shell of dome that its legs are loaded on the ball bearings, there is always the possibility to control and manipulate the arc dome. Also, there are other reasons for applying this type of arch in the dome such as relatively high prone of arc and low thrust at the abutments, loading capacity, and the beauty of its shape and its composition with Ogun curve in domes with Ogun (figure 1).

![Figure 1. Shabdari Arch [9: 40]](image)

One of the most famous buildings where the geometry of slow Shabdari arch is used in construction of its dome is Jame Mosque of Yazd that is an architecturally distinguished works of eighth and ninth centuries AD. Dome structure of this mosque is continuous double-shell and hollow (or cashier). Ahiyaneh and Nari dome are domes with slow Shabdari arch. General view of dome and motifs used in tiling under the dome are depicted in Figure 2 and 3, respectively.

![Figure 2. The dome and part of the south porch of Jame Mosque of Yazd [10]](image)
4. Parametric design

In complex planning, drawing inspiration from the natural patterns is very important, especially when we want to use these patterns in architectural design. By looking at the most natural patterns, the final form of the principle of birth can be found from tiny structures of its constituent. The concept that now is mentioned as parameterization in architecture.

Parametric design or in other words, grammar of geometric shapes is a new branch of computer sciences which aims to discover and apply mathematical and logical relationships between facts and figures on the one hand and shapes on the other hand. In fact, with the discovery of the relationships, the language and grammar of the connection of shapes with the digital world of computer are defined.

In the usual process of architectural design, often when the concept of design is drawn and modeled, certain operations that are uniform and repetitive occur. But, since traditional CAD tool is based on geometric objects, creating a change in design needs to change all related components in order to create a proper drawing. In parametric design which is a method to connect the dimensions and variables to the geometry, reform of design is significantly done faster than the time required for redrawing in traditional CAD systems. Parametric editing is done by creating scripts or changing the dimensions manually in digital model. In fact, the term of parametric design has a meaning far more complicated than using a computer instead of manually drawing. However, there are other related terms and synonyms that are used in the same sense like generator design, computational design, design with the help of computer, digital design.

Parametric design and its application in architecture can be investigated from two general perspectives. At first view, creativity and innovation can be expected from the computer by using parametric design and the forms and roles can be achieved that are beyond our expectation. In other words, computer is used as an Intelligent Designer in the design process and the designer will be able to decide to choose the best design. In the second view, the purpose of using parametric design is to create various designs by changing its geometric parameters. Thus, the geometric parameters of the design can accept distinct values in different parts of it to cause a gradual change in the motif and to avoid the repetition of the same components in the model and motif. Parametric modeling of a variety of forms and structures in existing software is based on two general methods of textual programming languages based textual and graphical programming languages. Textual programming languages use symbolic display such as texts and numbers to describe and execute operations on the existing drawings. Among the modeling software that work based on the textual programming languages can be pointed to the Formin software as well as all CAD scripting languages like Rhino Script and VBA. While the graphic or visual programming languages use analog display in which images (Icons) are indirectly applied for representation and changing drawings. Grasshopper is an example of this category. These environments provide the possibility to visually describe the relationships between the data without the need for writing code. Graphical programming languages,
known as geometric programming, allows the user to create programs by displacement and putting together the components of program in graphical form instead of writing code. Besides the normal geometric display, these packages have a graphical representation window, similar to programming codes in which the user can change the status quo by using analog display. At the same time, the user receives the results by viewing geometric model window. Also, when the user establishes a wrong connection between the elements, their color immediately becomes distinctive. While in scripting languages user often realizes his mistake during the failure and in useless code. Visual programming environments in the CAD software packages, through the simultaneous production of parameter range, are very effective for shape-finding. Many of these environments can be used without writing any code. However, in complex models, the capability of graphic programs can be promoted by scripting [13:121].

5. Reproduction of geometric patterns in parametric software of Grasshopper

Creativity and innovation can be expected from the computer by using parametric design and the forms and motifs can be achieved that are not already existed in our minds. These motifs can be used in various components of the architecture. Geometric parameters of a motif can have distinct values in its different parts, resulting in the emergence of a gradual change in motif and prevents the repetition of same components in that pattern and motif. Thus, by having the algorithm or formula of geometric motif production, design can be produced by changing various parameters of dozens and perhaps hundreds of different options. In this paper, to model the Iranian-Islamic geometric motifs as constituent structure of a dome with slow Shabdari arch space, a graphical programming environment with an emphasis on grasshopper parametric software is used. A variety of domes can be produced by using the algorithm of a sample of girih and by varying the amount of each of the variables. In modeling this type of geometric motifs in the software, the most important step is the detection of iterative base module on the surface of motif.

One of the best ways to generate the algorithm of geometric motifs is the utilization of the method of drawing radial infrastructural network that is the most common method of drawing Girih in the history books as well as the late Persian books. In this method, at first, the infrastructural network of girih in quarter of a full field (repeating or contagious unit) of girih is depicted. Then, by drawing three-quarters of this girih, with two successive imaging operations, its base module is obtained. Finally, by repeating the base module the girih can be expanded to the desired extent. The above-mentioned operations are done for each of the five widely used girihs shown in Table 1, and the base module of each of them is created in grasshopper software. The base module, full field and reproduction algorithm of parametric of mentioned girihs in grasshopper software are provided in Table 2-6, respectively.

One of the capabilities of grasshopper software is processing image of any pattern or specific shape on the quite arbitrary surface or volume. Thus, after the reproduction of each of the desired girihs, base module of each of them is depicted on slow Shabdari arch. The image of plan and view of arch after the image processing stage are shown in Table 7. Now we discuss the parametric concept of the problem. As previously stated, in order to create a parametric geometry, the designer should provide his preferred form through a series of relationships and parameters. The main reason for this is that the designer, in addition to creating his primary desired design, could also be able to achieve a series of other designing options by changing productive parameters of form. These alternative options are the same plans in which the computer appears in the role of a designer helper by focusing on the concept of making parametric. In this paper, the effect of variation of the parameters forming the base module of five girih is investigated. An important point that should be noted is that in grasshopper software, a change in the amount of converter parameters of the base form not only varies the module, but also updates all forms dependent on the module which here refer to processing the image of motifs on slow Shabdari arch.

| Table 2. Parametric design of Hashte-Chahar-Lengeh Girih[14] |
The base module

A full field of drawing

Table 3. Parametric design of Hashte-Panj Girih[14]

Algorithm of Girih reproduction and its image on any desired surface in Grasshopper software

Table 4. Parametric design of Hashte-Bazoubandi Girih[14]

Algorithm of Girih reproduction and its image on any desired surface in Grasshopper software

Table 5. Parametric design of Chahar-Lengeh-AlmasTarash Girih[14]
In this paper, the issue of parametric design and its effect on the final form of slow Shabdari arch are investigated in two following general categories:
- Changing the number of base modules in form of the final image without changing the form of module (making delicate the network).
- Changing the parameters of the production module and thereby, creating secondary forms. (Without changing the condensation of network)

In the first issue, the algorithm of image of base modules in arches of Table 7 has been linked to the dissemination coefficients of m and n that indicate the number of modules’ proliferation in horizontal and vertical direction of dome respectively. In this case, the aim is to create delicate networks based on the form of constant base module. In the other words, in this part, the final geometry of the base module depicted on the slow Shabdari arch remains fix, but the size of the productive base modules and thereby, their productive members become small or large in the real model. The importance of this issue becomes evident when the structural and constructional tips and considerations to be taken into account. In structural considerations, for such a dome which is displayed in the form of single-layer space structure, members are placed under
the shear fields, bending and torsional moment. Therefore, the maximum length of members is limited by the complete impact of the limitation of the amount of bending and torsional stresses and achieving any degree of sparseness in the ultimate form of architecture is not possible. Also, minimum length of member will be quite impressed by the number of connections used.

Table 7. Domes of slow Shabdari arch constructed by Islamic-Iranian Geometric motifs

<table>
<thead>
<tr>
<th>Side View</th>
<th>Plan</th>
<th>Girih</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Hashte-Chahar-Lengeh</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hashte-Panj</td>
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<tr>
<td></td>
<td></td>
<td>Hashte-Bazoubandi</td>
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<tr>
<td></td>
<td></td>
<td>Chahar-lengeh-almasTarash</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Table Khofte Rasteh</td>
</tr>
</tbody>
</table>

As a general rule, balance should be established in space structure between the maximum and minimum length of members on one hand, and on the other hand the number of connections. Changing the variables m and n to create different arches on the basis of base module of eight four-doublet motifs is provided in Figure 4. In the second case, initially, the parameters used in the algorithm of each base module are dependent on the dissemination coefficients i and j. In this case, the aim is to create new modules based on Islamic-Iranian geometric motifs. The point that should be noted is that by changing the parameters i and j, the form of base modules and thereby, its depicted form on arch, the primary form of motifs have not geometric fit with it. Therefore, some researchers and architects who emphasize on the originality of knit and the maintenance of its geometric proportions at each level or volume, do not consider the forms created by the second part of the proposed parametric issues as the motifs. The effect of changing the parameters i and j in the final form of created arch based on the base module of motif of eight four-doublet is provided.
in Table 8. Creating different modes by changing the parameters forming the base module of eight four doublet Girih By looking at Table 8, it can be seen that in parametric design of the second issue, not only varied forms have been created, but also in some cases, the dome of slow Shabdari arch has been converted to common domed space structures. For example, in the case of $i=j=0$ lamella dome and in the case of $i=0, j=1$ ribbed dome is produced.

Figure 4. Creating different modes by changing the number of modules in horizontal and vertical direction

Table 8. Domes of slow Shabdari arch constructed by Islamic-Iranian Geometric motifs
6. Conclusion

In this paper, parametric modeling of five widely used types of motifs of Iranian-Islamic geometry was provided in Grass Hopper software. The desired motifs are selected because of low condensate of members and the ability to construct appropriately as rod porter elements in the space dome of slow Shabdari arch. Then, the base module of parameterized motifs was depicted on the geometry of the slow Shabdari arch to create a domed space structure. Two different issues of parametric modeling were done for base module of one of the motifs of eight four-doublet Girih. Further, the effect of dissemination parameters of the base module on beauty and condensation of Shabdari arch, and the effect of the variation of pre-defined parameters in reproduction of the base module itself were considered. In the second style of parametric modeling, computer is appeared as designer helper and not merely as drawing and designing tool and therefore, presents unforeseen modules and more varied forms of the dome of slow Shabdari arch so that the results of computer simulations indicate that for some dissemination coefficients we can convert the pattern produced based on the motifs of Iranian-Islamic geometry in the dome of slow Shabdari arch to the common modules in space structures like lamella dome and ribbed dome.

References